

Soft equations of state for neutron-star matter ruled out by EXO 0748–676

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The interiors of neutron stars contain matter at very high densities, in a state that differs greatly from those found in the early Universe or achieved in terrestrial experiments¹. Matter in these conditions can only be probed through astrophysical observations that measure the mass and radius of neutron stars with sufficient precision². Here I report a determination of the mass and radius of the neutron star EXO 0748–676 that appears to rule out all the soft equations of state of neutron-star matter. If this object is typical, then condensates² and unconfined quarks¹ do not exist in the centres of neutron stars.

The neutron star source EXO 0748–676 has repeatedly shown thermonuclear X-ray bursts. It has also shown, first³ with EXOSAT and very recently⁴ with RXTE, a special type of thermonuclear X-ray burst that is strong enough to lift up the outer layers of the star. During these so-called photospheric radius expansion⁵ bursts, the radiation flux that emerges from the stellar surface is limited by the Eddington flux. It was, however, the detection⁶ of redshifted O and Fe lines by XMM-Newton from the surface of the neutron star that makes this low-mass X-ray binary unique in its class: here multiple phenomena have been observed from a single source that can be concurrently used to determine uniquely the equation of state of its dense interior.

The dependence of the three observable quantities—the Eddington limit, the redshift and the ratio F_{cool}/T_c^4 , which is closely related to the total emitting area—on the stellar properties is shown in Table 1 (F_{cool} is the thermal flux, and T_c is the colour temperature). Even though each of these measurements involve a different combination of the neutron-star mass, radius and distance, when all three observables are known, the expressions can be combined and inverted to yield a unique solution for the three stellar parameters, as shown in Table 2. This is also shown visually in Fig. 1, where the different constraints on the neutron star mass and radius imposed by each observable intersect at a unique set of values. One additional observable that, in principle, can tighten these constraints is the amount of rotational broadening in the redshifted lines. This leads to a direct measurement of the radius of the star⁷, with an uncertainty arising from the angle i that the observer makes with the rotational axis of the star (Fig. 1). For slowly spinning stars, however, other line broadening mechanisms, such as fine structure and Zeeman splitting, may dominate over the rotational broadening^{10,20}. Given the slow spin frequency of EXO 0748–676, rotational broadening alone cannot lead to a meaningful constraint on the stellar radius²⁰.

The only model parameters that are required in this solution are the colour-correction factor f_∞ and the electron scattering opacity κ_{es} . The colour correction factor f_∞ , for an observer at infinity, relates the colour temperature to the effective temperature, T_{eff} , of the star by $f_\infty \equiv T_c/T_{\text{eff}}$ and can be obtained by theoretical modelling of bursting neutron-star spectra. Here I use a fitting function that describes to an accuracy of 4% the results of recent model atmosphere

calculations⁸ for solar-abundance composition:

$$f_\infty = 1.34 + 0.25 \left(\frac{1+X}{1.7} \right)^{2.2} \left(\frac{T_{\text{eff}}^4 (\text{in } 10^7 \text{ K})}{g (\text{in } 10^{13} \text{ cm s}^{-2})} \right)^{2.2} \quad (1)$$

Here, g is the gravitational acceleration at the neutron-star surface. Note that f_∞ in Tables 1 and 2 corresponds to the colour-correction factor at the cooling tails of the X-ray bursts when the emerging flux is substantially sub-Eddington. At this limit, models of bursting atmospheres⁸ show that f_∞ asymptotes to a value of ~ 1.34 , practically independent of composition, temperature and stellar gravity, significantly reducing the model dependence of my results. For the electron-scattering opacity I use $\kappa_{\text{es}} (\text{in cm}^2 \text{ g}^{-1}) = 0.2(1+X)$, where $0 \leq X \leq 1$ is the hydrogen mass fraction. This is appropriate for the fully ionized neutron-star atmosphere during the radius-expansion episode. In this calculation, I allow for the entire range of values for X .

The main systematic uncertainty in my results arises from the assumption that the thermonuclear flash engulfs the entire star during the radius expansion and cooling phases of the bursts. There are a number of theoretical and observational arguments that support this assumption. First, numerical models⁹ of the spreading of thermonuclear bursts on the surface of rotating neutron stars show deflagration times that are $\ll 1$ s, which is shorter by orders of magnitude than the duration of the bursts. Second, the constraints¹⁰ on the magnetic field strength of EXO 0748–676, imposed by the lack of Zeeman splitting of the atomic lines, show

Table 1 | The three main quantities observed from EXO 0748–676 and their theoretical dependence on the neutron star properties

Observable	Measurement	Dependence on neutron-star properties
F_{Edd}	$(2.25 \pm 0.23) \times 10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}$	$\frac{1}{4\pi D^2} \frac{4\pi GMc}{\kappa_{\text{es}}} \left(1 - \frac{2GM}{Rc^2}\right)^{1/2}$
z	0.35	$\left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} - 1$
$F_{\text{cool}}/\sigma T_c^4$	$1.14 \pm 0.10 (\text{km kpc}^{-1})^2$	$f_\infty^4 \frac{R^2}{D^2} \left(1 - \frac{2GM}{Rc^2}\right)^{-1}$

The Eddington limit F_{Edd} , defined as the radiation flux at which the outward radiation force balances the inward gravitational force, is the limiting flux emerging from thermonuclear X-ray bursts with photospheric radius expansion. The measurements of the touchdown flux reported here were obtained by averaging the values determined recently with RXTE⁴ and earlier with EXOSAT³ observations, which are consistent with one another. The redshift z of O and Fe absorption lines in the X-ray burst spectra of EXO 0748–676 has been measured for the first time with XMM-Newton⁶. The ratio $F_{\text{cool}}/\sigma T_c^4$, where F_{cool} and T_c are the thermal flux and the colour temperature inferred from the X-ray burst spectra, respectively, asymptotes to a constant value during the cooling tails of the bursts. This ratio is directly related to the emitting surface area of the neutron star and was inferred from the RXTE data⁴ by taking the averages of $F_{\text{cool}}/\sigma T_c^4$ measured 7–15 s after the burst, ensuring that a constant ratio has been reached after the photospheric radius expansion but that the observed flux is still high enough for an accurate determination. The physical constants G and c are the gravitational constant and the speed of light, respectively, $\kappa_{\text{es}} = 0.2(1+X) \text{ cm}^2 \text{ g}^{-1}$ is the electron scattering opacity, X is the hydrogen mass fraction of the accreted material, f_∞ is the colour-correction factor, and D is the distance to the source. All the above expressions include minimal general relativistic corrections that are accurate in the slow rotation limit, which is appropriate for EXO 0748–676 given its 47-Hz spin frequency¹⁶.

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Table 2 | The neutron star properties that are obtained from the observations summarized in Table 1

Neutron-star property	Dependence on observables	Constraint
M	$\frac{f_{\infty}^4 c^5}{4\sigma_{\text{Kes}} \left(\frac{F_{\text{cool}}}{\sigma T_c^4}\right)} \frac{1-(1+z)^{-2}}{(1+z)^2} F_{\text{Edd}}^{-1}$	$2.10 \pm 0.28 M_{\odot}$
R	$\frac{f_{\infty}^4 c^3}{2\sigma_{\text{Kes}} \left(\frac{F_{\text{cool}}}{\sigma T_c^4}\right)} \frac{1-(1+z)^{-2}}{(1+z)^3} F_{\text{Edd}}^{-1}$	$13.8 \pm 1.8 \text{ km}$
D	$\frac{f_{\infty}^2 c^3}{2\sigma_{\text{Kes}} \left(\frac{F_{\text{cool}}}{\sigma T_c^4}\right)}^{1/2} \frac{1-(1+z)^{-2}}{(1+z)^2} F_{\text{Edd}}^{-1}$	$9.2 \pm 1.0 \text{ kpc}$

The neutron-star mass, radius and distance are uniquely determined from the Eddington luminosity, the redshift, and the ratio $F_{\text{cool}}/\sigma T_c^4$ as shown in the middle column, given a model for the neutron-star atmosphere that determines the colour-correction factor and a measurement of the hydrogen mass fraction. The last column shows the minimum values for the mass, radius and distance to the neutron star for any possible value of the colour-correction factor and the hydrogen mass fraction. The counter-intuitive dependence of the stellar properties on the observables arises from the combination of the expressions shown in Table 1. These expressions also show why a unique spectroscopic determination of the mass and radius of a neutron star, which is necessary to constrain its equation of state, can only be achieved when all three phenomena are observed from a single source, if the distance to the source is unknown.

that the magnetic field is dynamically unimportant and thus cannot inhibit the spreading of the thermonuclear flash over the entire surface.

On the observational side, further evidence for uniform emission from the entire surface as well as the constancy of the Eddington flux during the bursts is obtained from the study¹¹ of the peak luminosity of a large number (~ 70) of bursts from 4U1728–34, which shows that the peak flux is constant to within a few per cent in bursts separated by months. Indeed, a larger study¹² of peak fluxes of all thermonuclear burst sources in the RXTE catalogue also show a quantitatively similar result. Second, in the cooling tails of thermonuclear bursts, the observed ratio F_{cool}/T_c^4 asymptotes to a constant and reproducible value between bursts, strongly indicating that the entire surface of the neutron star emits uniformly in the cooling tails^{13,14}.

Even though flux oscillations of amplitude $\leq 10\%$ have been seen

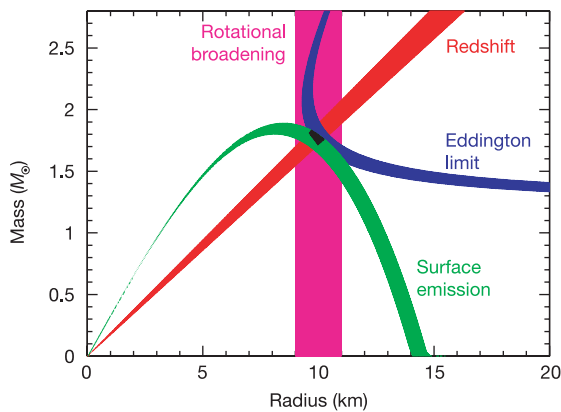


Figure 1 | Four complementary methods to determine the mass and radius of a neutron star. Shown are the contours on the mass–radius plane of neutron stars imposed by the measurement of the Eddington flux during photospheric radius expansion bursts (blue), the ratio F_{cool}/T_c^4 of the surface emission that asymptotes to a constant during the cooling tail of a thermonuclear burst (green), the redshift of atomic absorption lines observed during the burst (red), and the broadening of such lines due to the rotation of the star (magenta) for a hypothetical star with $M = 1.8 M_{\odot}$ and $R = 10 \text{ km}$. The second quantity, obtained from the thermal flux F_{cool} and the colour temperature T_c of the burst spectrum, is closely related to the total emitting area from the stellar surface when the nuclear burst has engulfed the entire star. The widths of the contours correspond to a hypothetical 10% uncertainty in each measurement. The uncertainty in the redshift measurement can be negligible if a grating spectrometer is used, as in the case of EXO 0748–676. The black-shaded area is the intersection of the four contours and corresponds to the uncertainty in the measurement of the true mass and radius of the neutron star.

in some thermonuclear bursts and are thought to be caused by modes excited on the neutron-star surface during the bursts¹⁵, their presence does not increase the uncertainties reported here. Burst oscillations have never been observed during the radius expansion phases of the bursts but only in the rise phase and cooling tails, thus not affecting the determination of the Eddington limit from observations. In addition, the low amplitudes of the oscillations during the cooling tails introduce uncertainties that are smaller than both the systematic and statistical uncertainties (10%) allowed for in my calculations. This is especially true for the burst oscillations in EXO 0748–676, which are very weak, with an average amplitude of 3% (ref. 16).

A final consideration about the Eddington limit is related to the point during a radius expansion burst at which this quantity is measured. The relevant measurement here is the so-called touchdown flux, which is the Eddington limit at the point in the burst when the photosphere returns to the actual radius of the star¹⁴. The flux at the touchdown point cannot be smaller than the Eddington flux because, if the radiation support of the atmosphere were suddenly removed, the photospheric radius would return to the actual size of the star within a free-fall timescale, which is $\sim 1 \text{ ms}$ for a neutron star. Instead, an adiabatic return to the stellar surface, at timescales of a few seconds, is observed in every radius expansion burst, indicating that the emerging flux traces the Eddington limit all the way to the touchdown point.

The only unknown property of the binary system that affects the mass and radius measurements is the hydrogen fraction X of the accreting material. Further observations of the elemental abundances of the accretion flow at long wavelengths could greatly reduce this uncertainty. However, even when I consider the most extreme value of $X = 0.7$, I can obtain lower limits on the mass and radius as follows:

$$M \geq 2.10 \pm 0.28 M_{\odot} \quad \text{and} \quad R \geq 13.8 \pm 1.8 \text{ km} \quad (2)$$

which is shown in Fig. 2. For smaller values of the hydrogen mass fraction, that is, for a helium-rich companion which can be expected

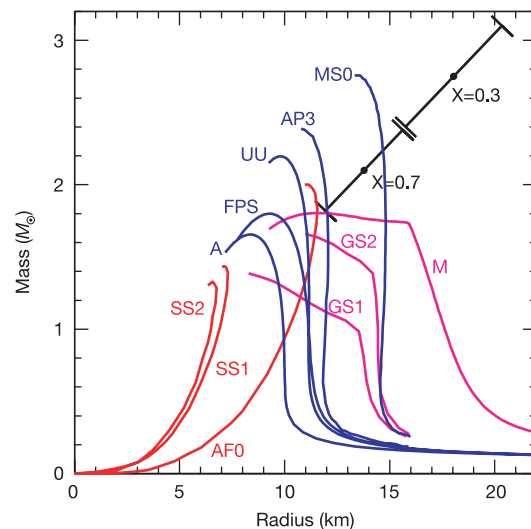


Figure 2 | The constraints on neutron star equations of state imposed by observations of EXO 0748–676. The predicted mass–radius relations for a number of representative equations of state of neutron stars without condensates (blue), with condensates (magenta), and for strange stars (red)^{17–19}. The curve labels and the corresponding references can be found in refs 17 and 18. The two 1- σ error bars correspond to the measurement of the mass and radius of EXO 0748–676 from three independent observations (equation (2)) for different values of the hydrogen mass fraction X of the accreted material. The measurement for $X = 0.7$ corresponds to the minimum allowed values for the mass and radius of the neutron star. Only the stiffest equations of state are consistent with this measurement.

in a small-orbit binary such as EXO 0748–676, the values of M and R are even higher. It is clear from Fig. 2 that the unknown value of X is by far the dominant systematic uncertainty in my result. Also note that even though separate uncertainties on mass and radius are quoted, these are not independent and do not form an error ellipse because the mass-to-radius ratio is fixed by the redshift measurement with a negligible error.

Because I treat distance as an independent variable, I can use the same three measurements to determine a lower limit on the distance to the source, $D \geq (9.2 \pm 1.0)$ kpc. This is a significant advantage of my method: The equations in Table 2 show that if the redshift is measured, the mass and radius measurements are independent of the distance and vice versa. Alternatively, if there is a direct measurement of the distance, for example, for a source in a globular cluster, then only two spectroscopic measurements are sufficient to determine the mass and radius of the neutron star.

The mass–radius measurement presented here does not suffer from uncertainties related to magnetic and centrifugal support that can complicate the calculation of neutron-star structure given an equation of state because of the slow rotation and the low magnetic field of EXO 0748–676. The large size and the high mass of this neutron star impose stringent constraints on the equation of state of matter at supernuclear densities (see Fig. 2). Even the lowest allowed value for the mass and radius can be obtained by only the stiffest equations of state. In particular, equations of state with condensate interiors predict large-radius, low-mass neutron stars (Fig. 2) that are mostly excluded by my measurements. On the other extreme of the mass–radius diagram, self-bound bare strange-matter stars have mostly small masses and radii, also inconsistent with the mass and radius of EXO 0748–676. The more conventional equations of state with neutron–proton compositions are more likely to explain the set of observational properties of EXO 0748–676 presented here. I therefore argue that hadrons and not deconfined quarks represent the ground state of matter.

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